Factoring Polynomials

These notes are intended as a summary of section 1.2 (p. 15 – 19) in your workbook. You should also read the section for more complete explanations and additional examples.

Factoring

A **factor** is any binomial that divides evenly into a polynomial, with no remainder. In this lesson, we will discuss two major concepts:

1. How can we determine if a binomial is a factor, without dividing?

2. If a binomial isn't a factor, how can we determine the remainder without dividing?

For each polynomial below, divide to determine whether the given binomial is a factor. Each binomial has the form $x - a$. Then evaluate the polynomial when $x = a$.

a) $3x^2 - 2x - 1$ $\qquad$ $x - 1$

b) $3x^3 - 8x^2 - x - 2$ $\qquad$ $x - 3$

c) $2x^4 - x^3 - 17x^2 - 11x + 6$ $\qquad$ $x + 2$

In each of the above examples, the remainder (found by dividing) and the value of the polynomial when $x = a$ were the same. This is called the **remainder theorem**.

Remainder Theorem

When a polynomial $P(x)$ is divided by a binomial, $x-a$, the remainder is $P(a)$.

Example 1 (sidebar p. 17)
Determine the remainder when $2x^4-5x^3-5x^2+5x+3$ is divided by each binomial.

a) $x-3$

b) $x+2$

In Example 1a, the remainder was zero. This means the divisor, $x-3$, is a factor of the polynomial. This special case of the remainder theorem is called the **factor theorem**.

Factor Theorem

A binomial, $x-a$, is a factor of the polynomial $P(x)$ if $P(a)=0$.

Example 2 (sidebar p. 18)
Which binomials are factors of $x^3-6x^2+5x+12$?

a) $x+1$

b) $x-3$

c) $x-4$

d) $x+4$

From Example 2, there are three factors of $x^3-6x^2+5x+12$: $x+1$, $x-3$, and $x-4$. The product of these factors is the original polynomial.

$$(x+1)(x-3)(x-4)=x^3-6x^2+5x+12$$

Notice that the constant term in each binomial is a factor of the constant term in the polynomial. That is, each of –1, 3, and 4 is a factor of 12. This leads to the **factor property**.

Factor Property

If $x - a$ is a factor of a polynomial, then a is a factor of the constant term in the polynomial.

Example 3 (sidebar p. 19)
Factor fully: $3x^3 - 4x^2 - 5x + 2$

Steps:

1. Use the factor theorem to find one factor.
2. Use synthetic division and write the division statement.
3. Factor the quadratic (if possible) or repeat steps 1 and 2 as necessary.

Homework: #4 – 8, 9 (factor completely), 10 – 12 in the exercises (p. 20 – 25). Answers on p. 26.